

Richardson's length measure (1961)

$$\Sigma(L) = L^{-\alpha}$$

Values of α for selected boundaries

Australia	0.13
South Africa	0.02
Germany	0.15
Britain	0.25
Portugal	0.14

Mandelbrot's re-expression of length (1966)

$$r = \left(\frac{1}{N}\right)^{\frac{1}{D}} \quad D > 1.0 \text{ for lines}$$

r = increase in total length
 N = unit of measure

Solving for D :

$$\ln(r) = \ln\left[\left(\frac{1}{N}\right)^{\frac{1}{D}}\right]$$

$$\ln(r) = \frac{1}{D} * \ln\left(\frac{1}{N}\right)$$

$$D * \ln(r) = -\ln(N)$$

$$D = \frac{-\ln(N)}{\ln(r)}$$

$$D = \frac{\ln(N)}{\ln\left(\frac{1}{r}\right)}$$

For a Koch curve:

$$D = \frac{\ln(4)}{\ln\left(\frac{1}{1/3}\right)}$$

$$D = \frac{\ln(4)}{\ln(3)}$$

$$D = 1.2618$$

