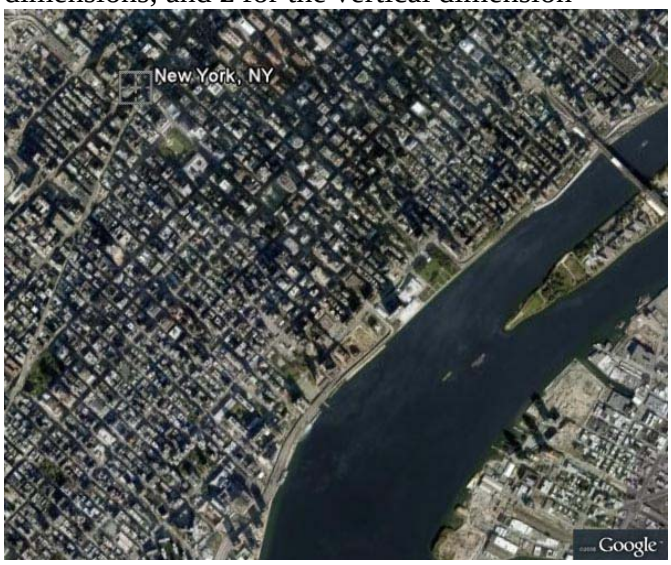
Math, Units, Physical Quantities

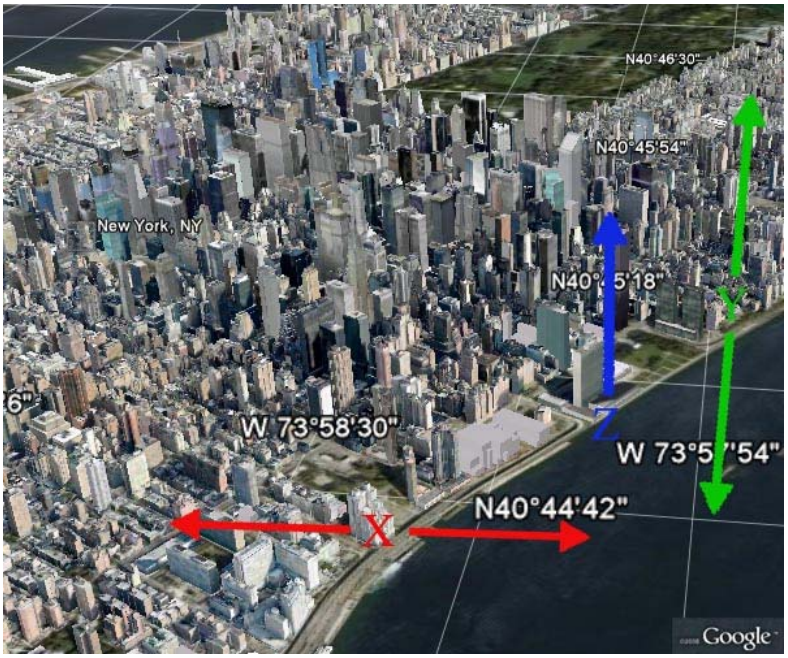
Use of math in spatial analysis helps us tremendously. For example, what would you do if you wanted to know how many birds are present in a certain wetland? Or, what would you do if you needed to figure out the rate of pollution change? Although computers can automate some of these tasks and computations for us we still need to know what formulas to use and what the answers indicate. The following sections provide a general overview of topics from mathematics.

Algebra:

- logical representation of numbers by lettered variables
- geographic variables are functions of space and time
    - space can represented by the **x, y** Cartesian coordinates for the horizontal dimensions, and **z** for the vertical dimension







- time is designated by the variable **t**

- a function (*f*) is where one variable varies according to another variable
  e.g., R= rainfall is a function of location (x, y), elevation (z) and time (t),
  therefore R= *f* (x, y, z, t)
- useful algebraic manipulations; equations can be simplified or reorganized by conducting the same operation to both sides of the equation:
  - if x/y = z/y ; multiplying both sides by y: $y \times x/y = y \times z/y$; then x= z
  - if x= z; dividing by x: $x/x = z/x$; then 1 = z/x
  - if $x= x^2/y$; then dividing by x; $x/x = x^2/yx$; then 1= x/y
- solving algebraic equations:
  - if n variables are unknown then n equations are required to solve for all variables
  - cross-multiplication is useful for solving one equation with three known and one unknown variables

  e.g., $\frac{d}{D} = \frac{1}{SF} \rightarrow \frac{d}{D} \times \frac{1}{SF} \rightarrow D \times 1 = d \times SF \rightarrow D \times 1 = d \times SF$
  $\downarrow$
  $D = d \times SF$

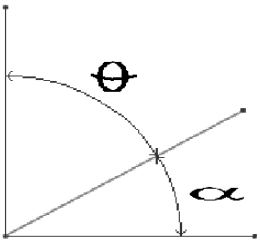
  d = map/image distance; D = ground distance; SF = Scale Factor

- linear equations:
  y = mx + b
  m = slope = y/x + b/x
  b = y - mx

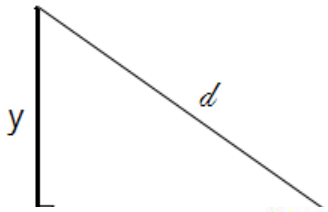
Geometry

- Geometry "is a part of mathematics concerned with questions of size, shape, relative position of figures and with properties of space." – Wikipedia
- Although we often think of the Earth as a perfectly round sphere, it is technically more of an ellipse. The field of geodesy or geodetics uses geometry to better understand the overall shape of the Earth.

Complementary angles:
$\theta = 90^o - \alpha$ and $\alpha = 90^o - \theta$

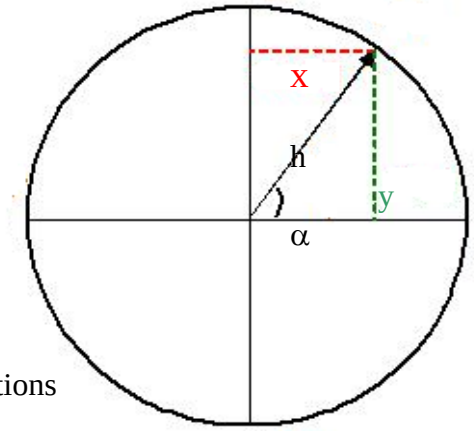


Pythagorean Theorem:
$d^2 = x^2 + y^2$ and $d = (x^2 + y^2)^{1/2}$
$x^{1/2} = \sqrt{x}$



Trigonometry:

Cosine: $\cos \alpha = x/h$
Sine: $\sin \alpha = y/h$
Tangent: $\tan \alpha = \sin \alpha / \cos \alpha = y/h \div x/h = y/x$
Cotangent: $\cot \alpha = x/y$
Cosecant = h/x
Secant = h/x
Arccos $(x/h) = \cos^{-1}(x/h) = \alpha$
Use tables, calculators, or computers to find trig functions



Calculus:

If $x = f(t)$
$\triangle x$ = discrete interval/increment of x (i.e. space)
$\triangle t$ = interval/increment of t (i.e. time)

dx when $\triangle x \rightarrow 0$ is called the derivative of x
$dx/dt = \triangle x/\triangle t \rightarrow 0$ = instantaneous velocity where
x= space and t= time
$dx/dt/dt = d^2x/dt^2$ = instantaneous acceleration where
x= space and t = time

$x = f(y, t)$
dx/dy, dx/dt
differential equation: e.g., $dx/dt + by = c$
partial differential equation: e.g., $\delta x/\delta t + \delta x/\delta y = 0$

Exponents:

Base 10	Exponential Notation
0	0.0
0 to 1	$1.0 \times 10^{\infty}$ to 1.0×10^{0}
1 to 10	1.0×10^{-0} to 1.0×10^{1}
10 to 100	1.0×10^{1} to 1.0×10^{2}

Exponent Laws:

$a^m \times a^n = a^{m+n}$	$x^{-1} = 1/x$	$x^{1/2} = \sqrt{x}$
$(a^m)^n = a^{mn}$	$x^{-2} = 1/x^2$	

Base e

$e = 2.718\ldots$ $\ln(e^x) = x$ $e^{\ln(x)} = x$

Base 2 (binary) – Used for computer data storage

The binary number system uses digits, 0 and 1, to represent a given value. To convert from binary to decimal notation begin with the rightmost digit and work to the left. This is because the right-side of the sequence represents the lowest number. Below are some common exponents that you might encounter when using the binary system.
$2^0 = 1$; $2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $2^5 = 32$; $2^6 = 64$; $2^7 = 128$; 28 = 256

1 byte = 2^8 e.g., 00000000 = 0 2^8 00000001 = 1 11111110 = 254 11111111 = 255

Units:

There are many different ways to measure the world and attributes. If we are looking at changes in vegetation we may want to measure time in years. However, if we are to consider the traffic patterns on I-8 we may want to consider using hours or minutes.

It is also valuable to know how to convert between units of measure. Not everyone will use the same measurements, for example one researcher could measure distance in meters and another in feet. For these researchers to be able to share data they have to be able to convert their data.

The four basic dimensions of physical mechanics are:

1. Mass (m)
2. Distance (L)
3. Time (t)
4. Temperature (T)

Other basic dimension which are less critical to our study of physical geography also exist, such as electrical charge and magnetism

Systems of unit measure include.

1. SI (System International), also called MKS and is the preferred system
2. British Gravitational
3. English Engineering
4. cgs (type of SI)

The basic SI units that correspond to the four basic dimensions are:

Units	**Dimensions**
Kilogram (kg)	Mass
Meter (m)	Distance
Second (s)	Time
Degree Kelvin (^{o}K)	temperature

Physical Quantities:

Physical Quantify	**Definition**	**Dimensional Units**	**SI Units**
Length	Linear distance	L	m
Area	2-D distance	L^2	m^2
Volume	3-D distance	L^3	m^3
Velocity	Distance per time interval	L/t or Lt^{-1}	ms^{-1}
Acceleration	Rate of velocity change	Lt^{-2}	ms^{-2}
Density	Mass per volume	ML^{-3}	Kgm^{-3}
Force	Mass time acceleration	MLt^{-2}	Kg^{-2}= Newton
Energy	Force over distance	ML^2t^{-2}	Nm = Joule (J)
Power	Energy per unit time	ML^2t^{-3}	Js^{-1}= Watt (W)
Power flux	Power per square area	Mt^{-3}	Wm^{-2}
Pressure	Force per square area	$ML^{-1}t^{-2}$	Nm^{-2}= Pascal (Pa) or 1×10^5 Nm^{-2}= bar

Solar Angles:

Zenith angle - an angular measurement from straight up (zenith) to a point in the sky. Zenith angle can be used along with azimuth to indicate the position of a star or other celestial body. Zenith angle is the complementary angle of the elevation (elevation = 90° - zenith).
- http://www.srrb.noaa.gov/highlights/sunrise/glossary.html#z

