Spatial Statistics and Analysis Methods

(for GEOG 104 class).

- Provided by Dr. An Li, San Diego State University.

Types of spatial data

- Points
  - Point pattern analysis (PPA; such as nearest neighbor distance, quadrat analysis)
  - Moran's I, Getis G*
- Areas
  - Area pattern analysis (such as join-count statistic)
  - Switch to PPA if we use centroid of area as the point data
- Lines
  - Network analysis

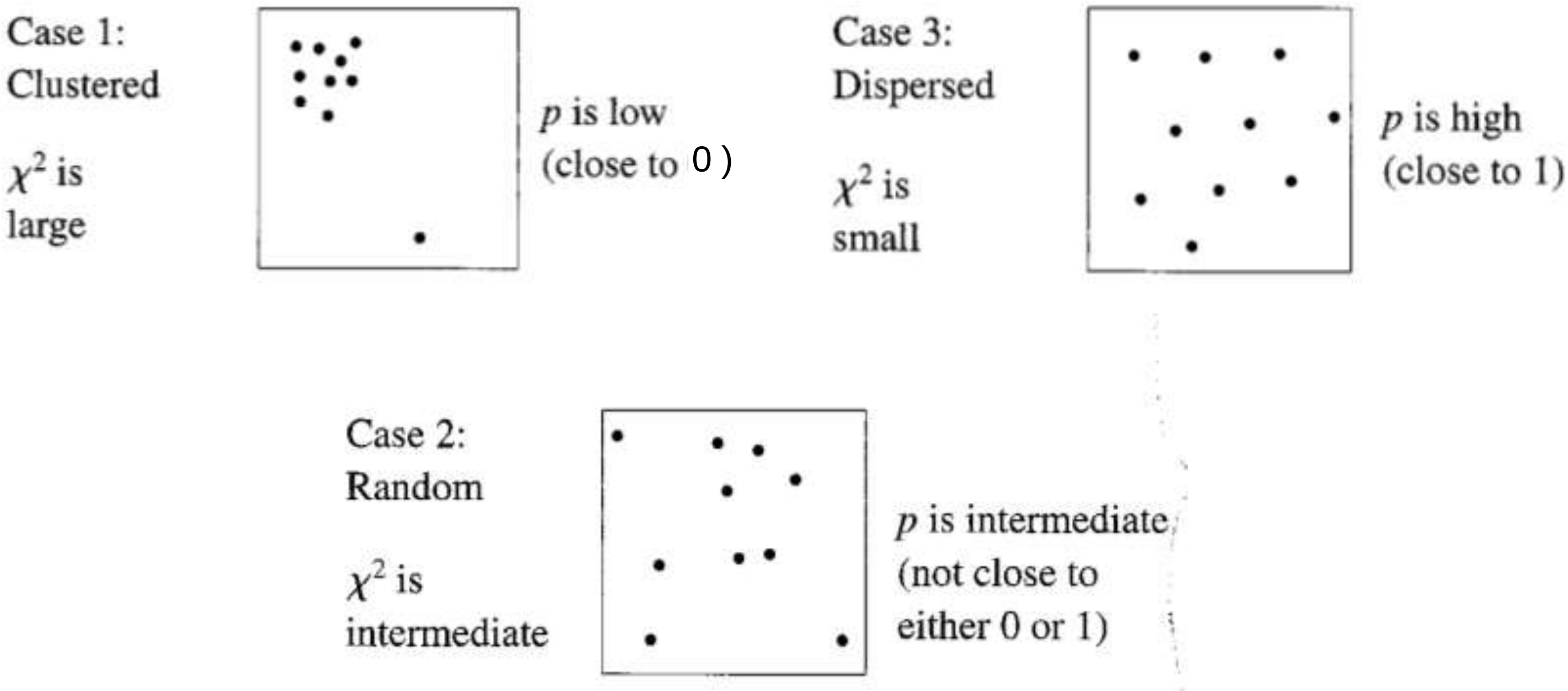
→Three ways to represent and thus to analyze spatial data:

Spatial arrangement

- Randomly distributed data
  - The assumption in "classical" statistic analysis
- Uniformly distributed data
  - The most dispersed pattern—the antithesis of being clustered
  - Negative spatial autocorrelation
- Clustered distributed data
  - **Tobler's Law** — all things are related to one another, but near things are more related than distant things
  - Positive spatial autocorrelation

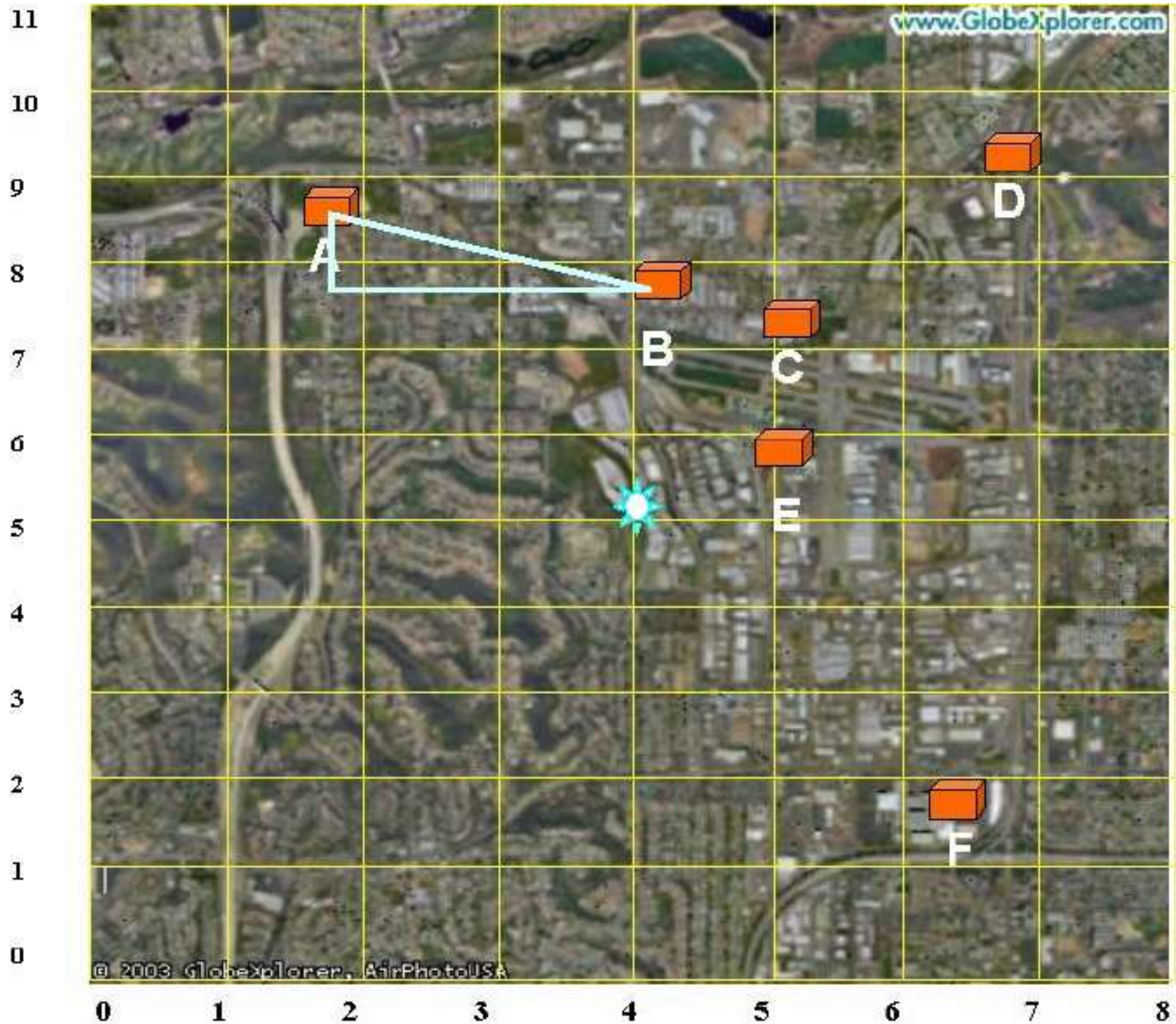
→**Three basic ways in which points or areas may be spatially arranged**

Spatial Distribution with ***p*** value



Nearest neighbor distance

- Questions:
  - What is the pattern of points in terms of their nearest distances from each other?
  - Is the pattern random, dispersed, or clustered?
- Example
  - Is there a pattern to the distribution of toxic waste sites near the area in San Diego (see next slide)? [hypothetical data]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- Step 1: Calculate the distance from each point to its nearest neighbor, by calculating the hypotenuse of the triangle:

$$NND_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

Site	X	Y	NN	NND
A	1.7	8.7	B	2.79
B	4.3	7.7	C	0.98
C	5.2	7.3	B	0.98
D	6.7	9.3	C	2.50
E	5.0	6.0	C	1.32
F	6.5	1.7	E	4.55
				13.12

$$\overline{NND} = \frac{\sum NND}{n} = \frac{13.12}{6} = 2.19$$

- Step 2: Calculate the distances under varying conditions
  - The average distance if the pattern were random?

$$\overline{NND_R} = \frac{1}{2\sqrt{Density}} = \frac{1}{2\sqrt{0.068}} = 1.92$$

Where density = n of points / area=6/88=0.068

  - If the pattern were completely clustered (all points at same location), then:

$$\overline{NND_C} = 0$$

  - Whereas if the pattern were completely dispersed, then:

$$\overline{NND_D} = \frac{1.07453}{\sqrt{Density}} = \frac{1.07453}{0.261} = 4.12$$

(Based on a Poisson distribution)

- Step 3: Let’s calculate the standardized nearest neighbor index (R) to know what our NND value means:

$$R = \frac{\overline{NND}}{\overline{NND_R}} = \frac{2.19}{1.92} = 1.14$$

= slightly more <u>dispersed</u> than random

Perfectly dispersed 2.15

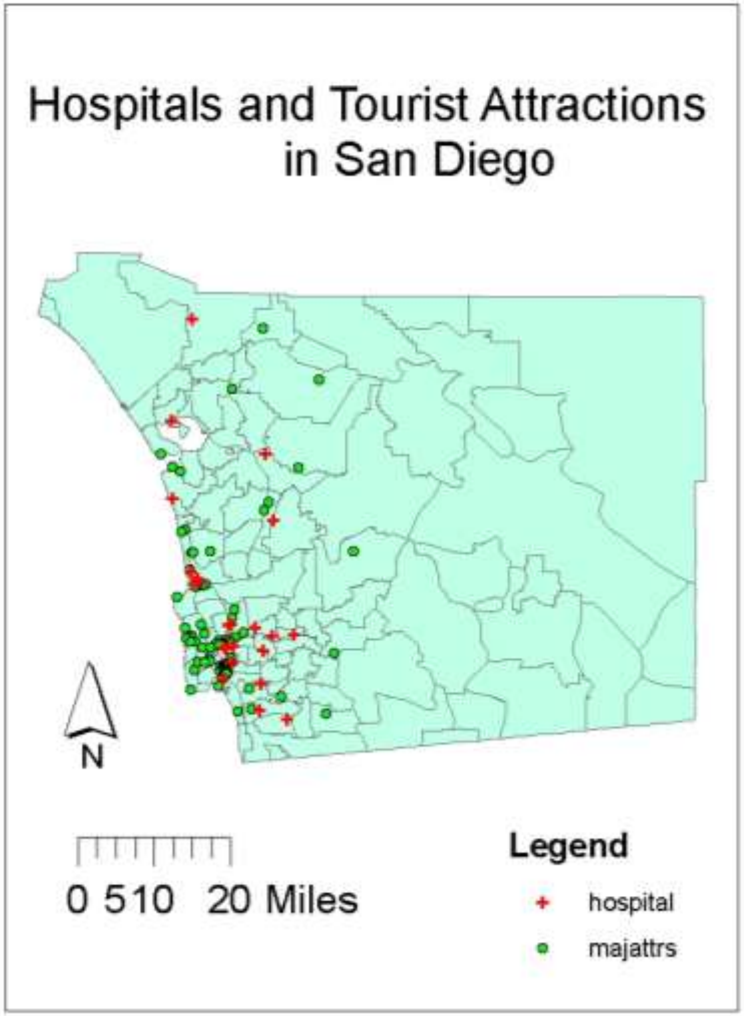
More dispersed than random

Totally random 1

More clustered than random

Perfectly clustered 0

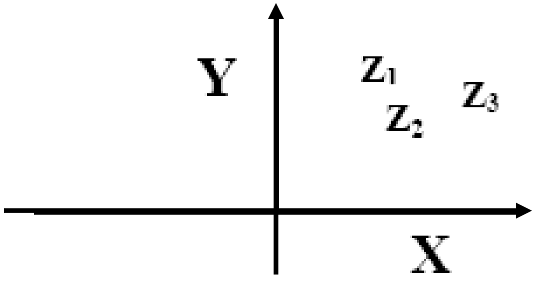
Hospitals & Attractions in San Diego



- The map shows the locations of hospitals (+) and tourist attractions (●) in San Diego
- Questions:
  - Are hospitals randomly distributed
  - Are tourist attractions clustered?

Spatial Data (with X, Y coordinates)

- Any set of information (some variable 'z') for which we have locational coordinates (e.g. longitude, latitude; or x, y)



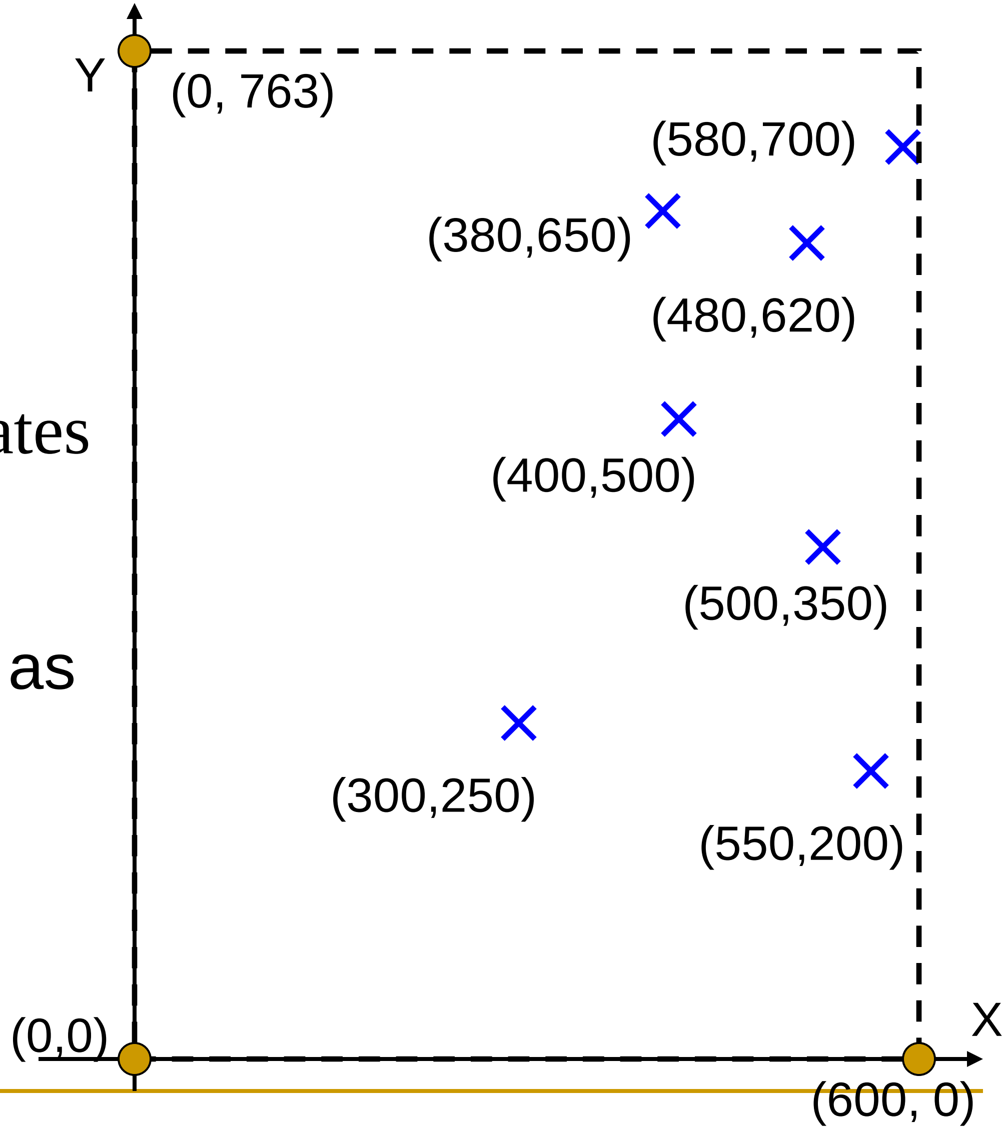
- Point data are straightforward, unless we aggregate all point data into an areal or other spatial units
- Area data require additional assumptions regarding:
  - Boundary delineation
  - Modifiable areal unit (states, counties, street blocks)
  - Level of spatial aggregation = scale

Area Statistics Questions

- 2003 forest fires in San Diego
- Given the map of SD forests
  - What is the average location of these forests?
  - How spread are they?
  - Where do you want to place a fire station?

What can we do?

- Preparations
  - Find or build a coordinate system
  - Measure the coordinates of the center of each forest
  - Use centroid of area as the point data



Mean center

- The mean center is the "average" position of the points
- Mean center of X: $\bar{X}_C = \frac{\sum x}{n}$

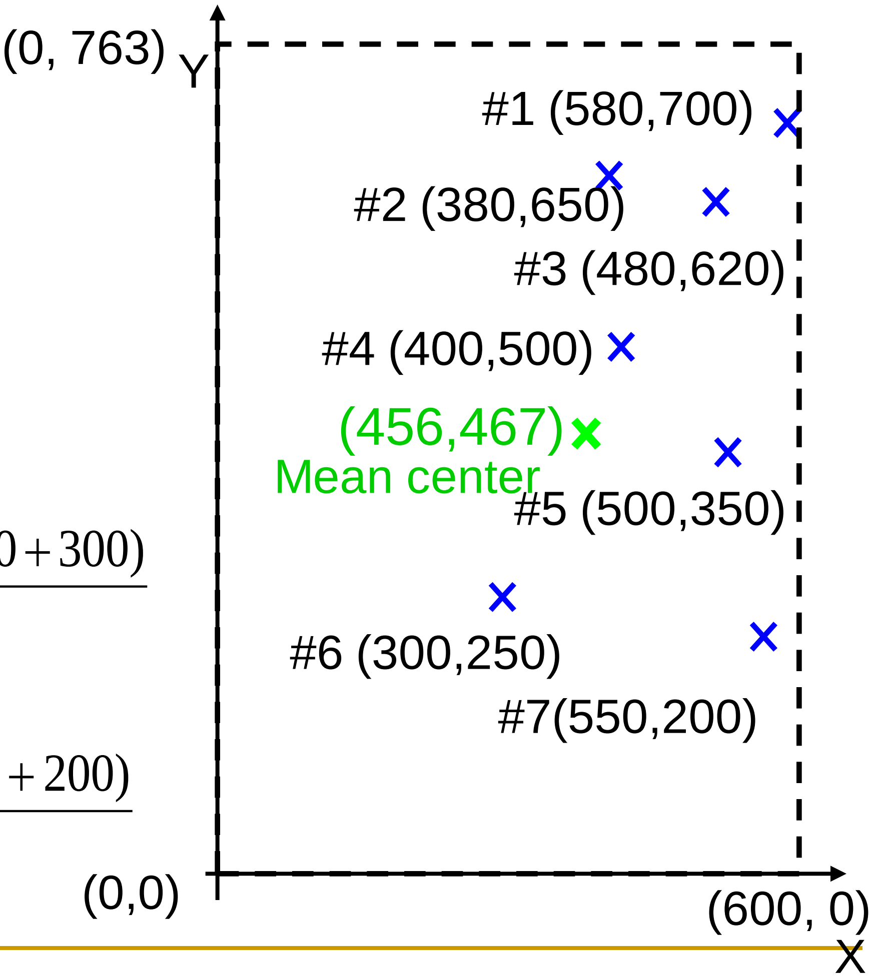
  Mean center of Y: $\bar{Y}_C = \frac{\sum y}{n}$

$$\bar{X}_C = \frac{(580+380+480+400+500+550+300)}{7}$$

$$= 455.71$$

$$\bar{Y}_C = \frac{(700+650+620+500+350+250+200)}{7}$$

$$= 467.14$$



Standard distance

- The standard distance measures the amount of dispersion
  - Similar to standard deviation
  - Formula

$$S_D = \sqrt{\frac{\sum (X_i - \bar{X}_c)^2 + \sum (Y_i - \bar{Y}_c)^2}{n}}$$ ← Definition

$$S_D = \sqrt{\left(\frac{\sum X_i^2}{n} - \bar{X}_c^2\right) + \left(\frac{\sum Y_i^2}{n} - \bar{Y}_c^2\right)}$$ ← Computation

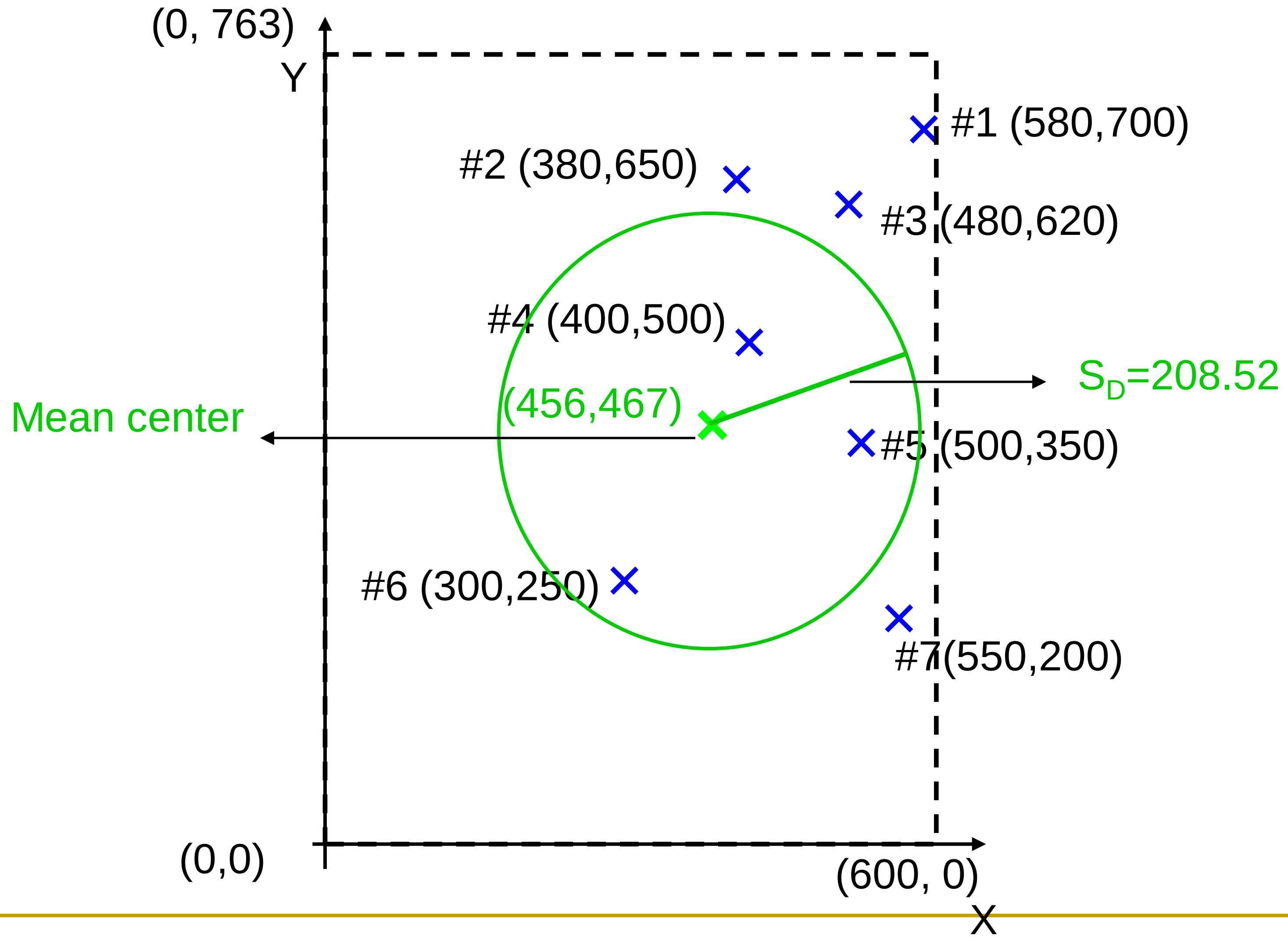
Standard distance

Forests	X	X^2	Y	Y^2
#1	580	336400	700	490000
#2	380	144400	650	422500
#3	480	230400	620	384400
#4	400	160000	500	250000
#5	500	250000	350	122500
#6	300	90000	250	62500
#7	550	302500	200	40000
	Sum of X^2	1513700	Sum of X^2	1771900
	$\bar{X}_C = 455.71$		$\bar{Y}_C = 467.14$	

$$S_D = \sqrt{\left(\frac{\sum X_i^{\,2}}{n} - \bar{X}_c^{\,2}\right) + \left(\frac{\sum Y_i^{2}}{n} - \bar{Y}_c^{\,2}\right)}$$

$$= \sqrt{\left(\frac{1513700}{7} - 455.71^2\right) + \left(\frac{1771900}{7} - 467.14^2\right)} = 208.52$$

Standard distance



Definition of weighted mean center standard distance

- What if the forests with bigger area (the area of the smallest forest as unit) should have more influence on the mean center?

$$\bar{X}_{wc} = \frac{\sum f_i X_i}{\sum f_i} \qquad \bar{Y}_{wc} = \frac{\sum f_i Y_i}{\sum f_i}$$

$$S_{WD} = \sqrt{\frac{\sum f_i (X_i - \bar{X}_{wc})^2 + \sum f_i (Y_i - \bar{Y}_{wc})^2}{\sum f_i}} \longleftarrow \text{Definition}$$

$$S_{WD} = \sqrt{(\frac{\sum f_i X_i^{\,2}}{\sum f_i} - \bar{X}_{wc}^2) + (\frac{\sum f_i Y_i^{\,2}}{\sum f_i} - \bar{Y}_{wc}^2)} \longleftarrow \text{Computation}$$

Calculation of weighted mean center

- What if the forests with bigger area (the area of the smallest forest as unit) should have more influence?

Forests	f(Area)	X_i	f_iX_i (Area*X)	Y_i	f_iY_i (Area*Y)
#1	5	580	2900	700	3500
#2	20	380	7600	650	13000
#3	5	480	2400	620	3100
#4	10	400	4000	500	5000
#5	20	500	10000	350	7000
#6	1	300	300	250	250
#7	25	550	13750	200	5000
$\sum f_i$	86	$\sum f_iX_i$	40950	$\sum f_iY_i$	36850

$$\overline{X}_{wc} = \frac{\sum f_i X_i}{\sum f_i} = \frac{40950}{86} = 476.16 \quad \overline{Y}_{wc} = \frac{\sum f_i Y_i}{\sum f_i} = \frac{36850}{86} = 428.49$$

Calculation of weighted standard distance

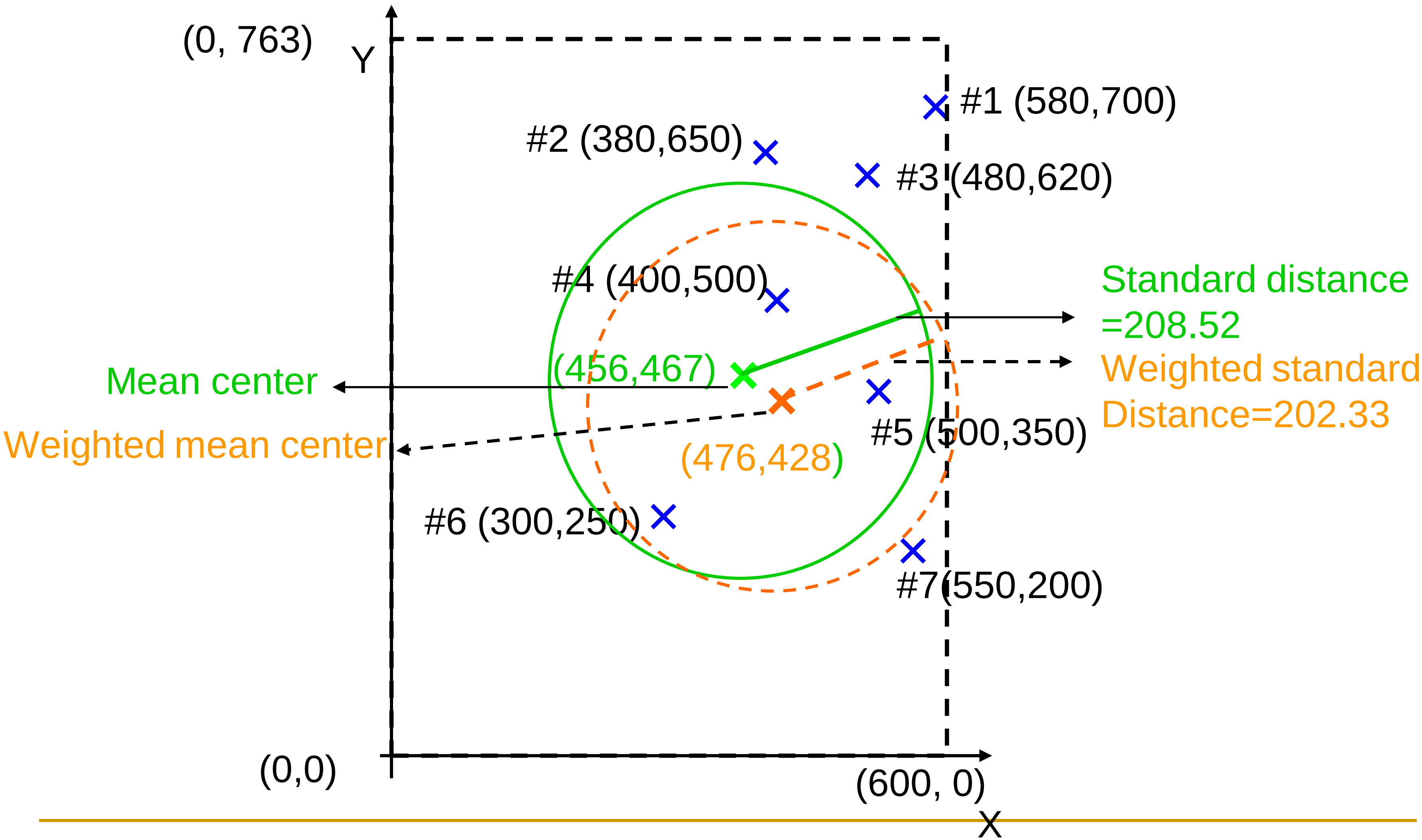
- What if the forests with bigger area (the area of the smallest forest as unit) should have more influence?

Forests	f_i(Area)	X_i	X_i^2	$f_i X_i^2$	Y_i	Y_i^2	$f_i Y_i^2$
#1	5	580	336400	1682000	700	490000	2450000
#2	20	380	144400	2888000	650	422500	8450000
#3	5	480	230400	1152000	620	384400	1922000
#4	10	400	160000	1600000	500	250000	2500000
#5	20	500	250000	5000000	350	122500	2450000
#6	1	300	90000	90000	250	62500	62500
#7	25	550	302500	7562500	200	40000	1000000
$\sum f_i$	86		$\sum f_i X_i^2$	19974500		$\sum f_i Y_i^2$	18834500

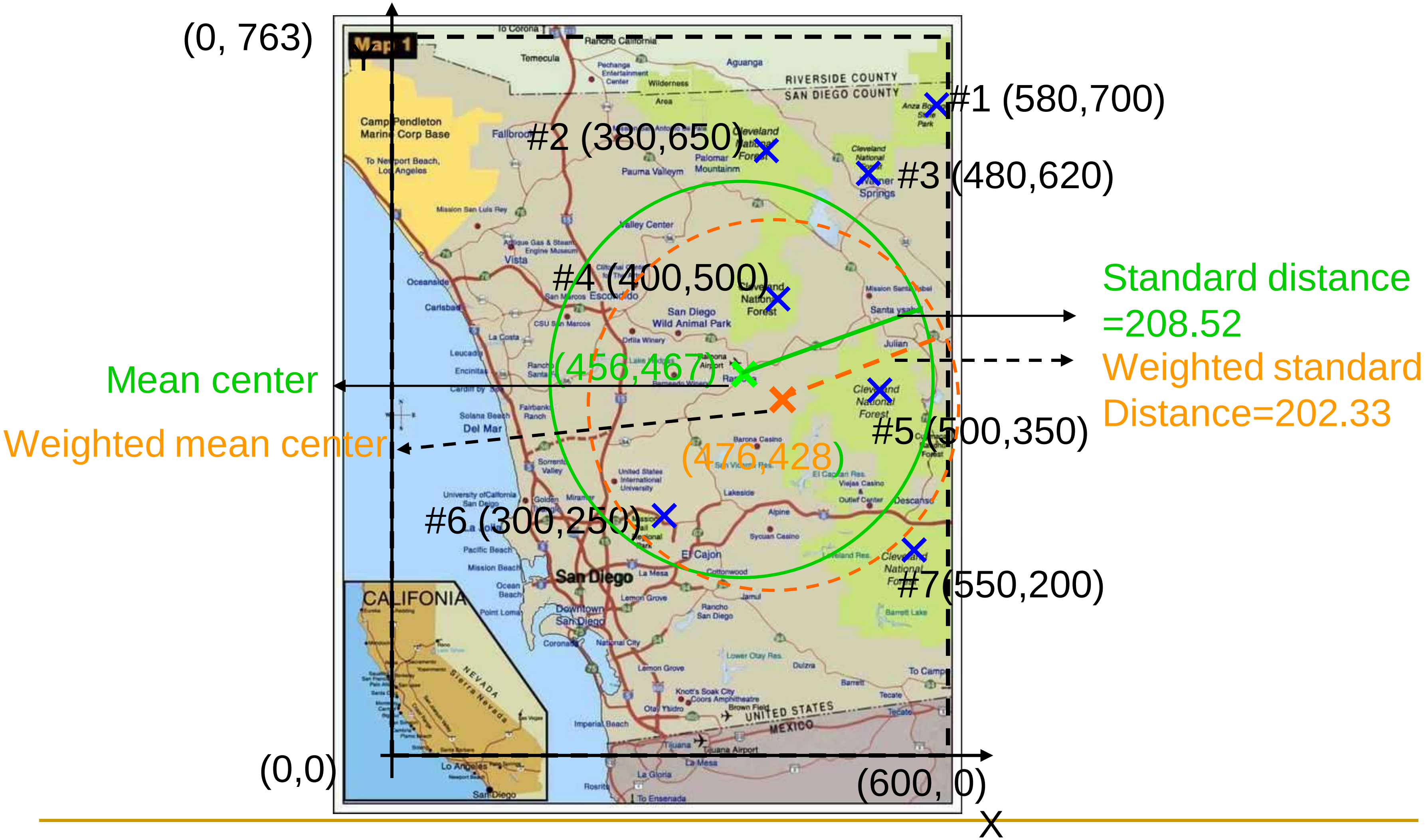
$$S_{WD} = \sqrt{\left(\frac{\sum f_i X_i^2}{\sum f_i} - \overline{X}_{wc}^2\right) + \left(\frac{\sum f_i Y_i^2}{\sum f_i} - \overline{Y}_{wc}^2\right)}$$

$$= \sqrt{\left(\frac{19974500}{86} - 476.16^2\right) + \left(\frac{18834500}{86} - 428.49^2\right)} = 202.33$$

Standard distance

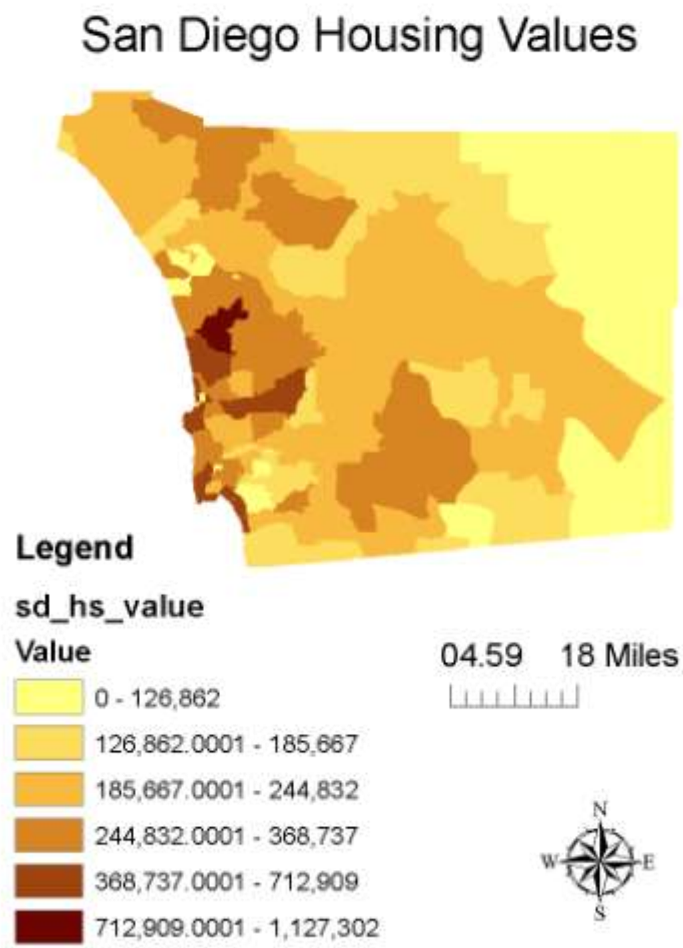


Standard distance



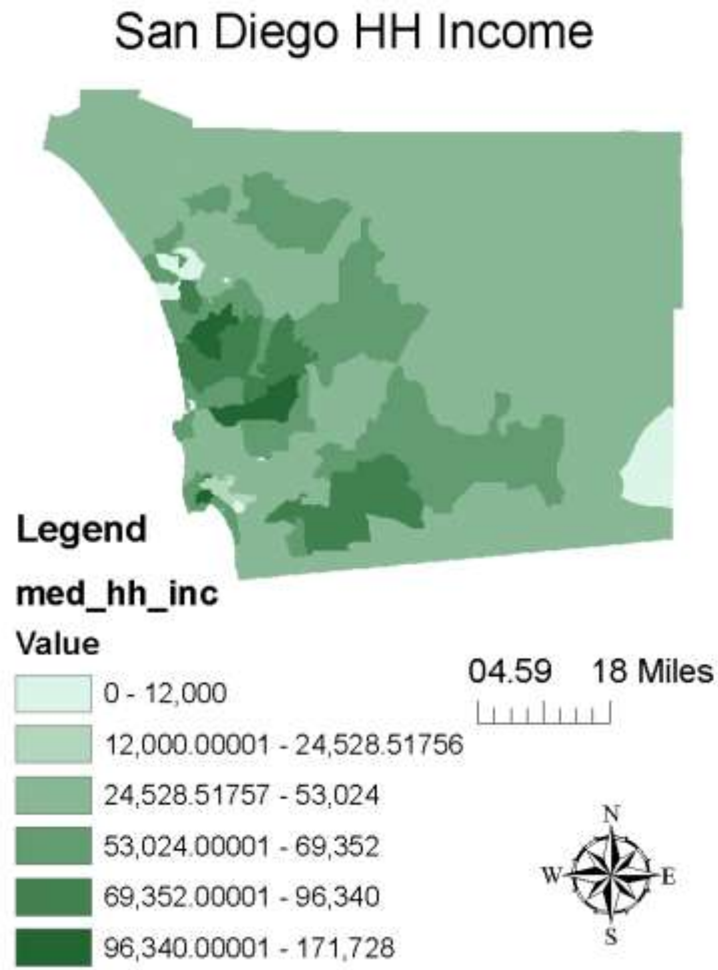
Spatial clustered?

- Given such a map, is there strong evidence that housing values are clustered in space?
  - Lows near lows
  - Highs near highs



More than this one?

- Does household income show more spatial clustering, or less?



Moran's I statistic

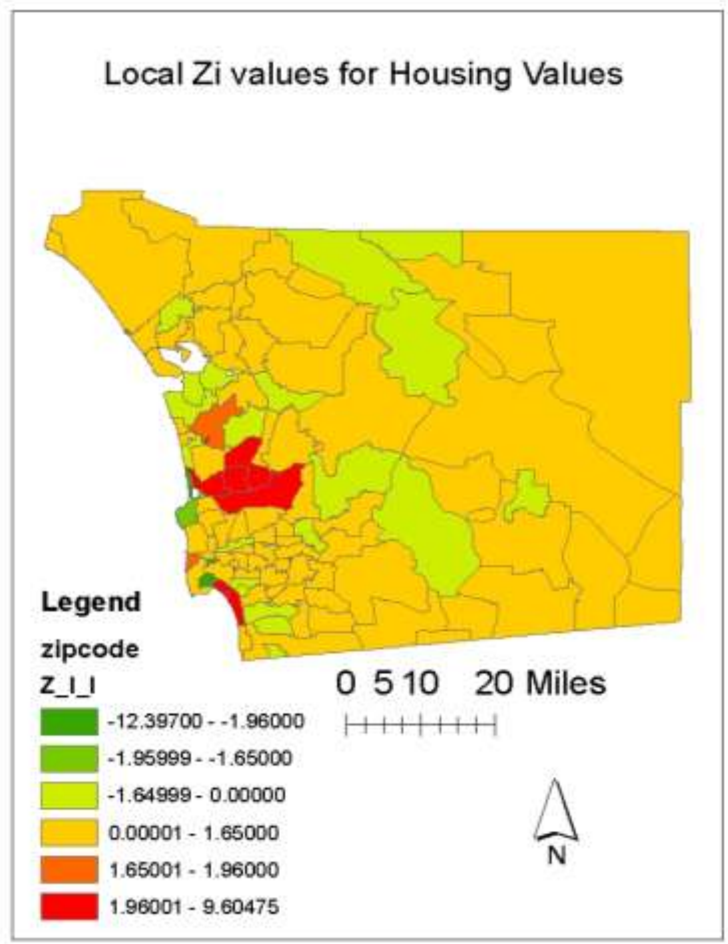
- Global Moran's I
  - Characterize the overall spatial dependence among a set of areal units

$$I = \left(\frac{n}{\sum_{i=i}^{n} \sum_{j=1}^{n} w_{ij}} \right) \left(\frac{\boxed{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \left(x_i - \bar{x}\right)\left(x_j - \bar{x}\right)}}{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2} \right)$$

Covariance

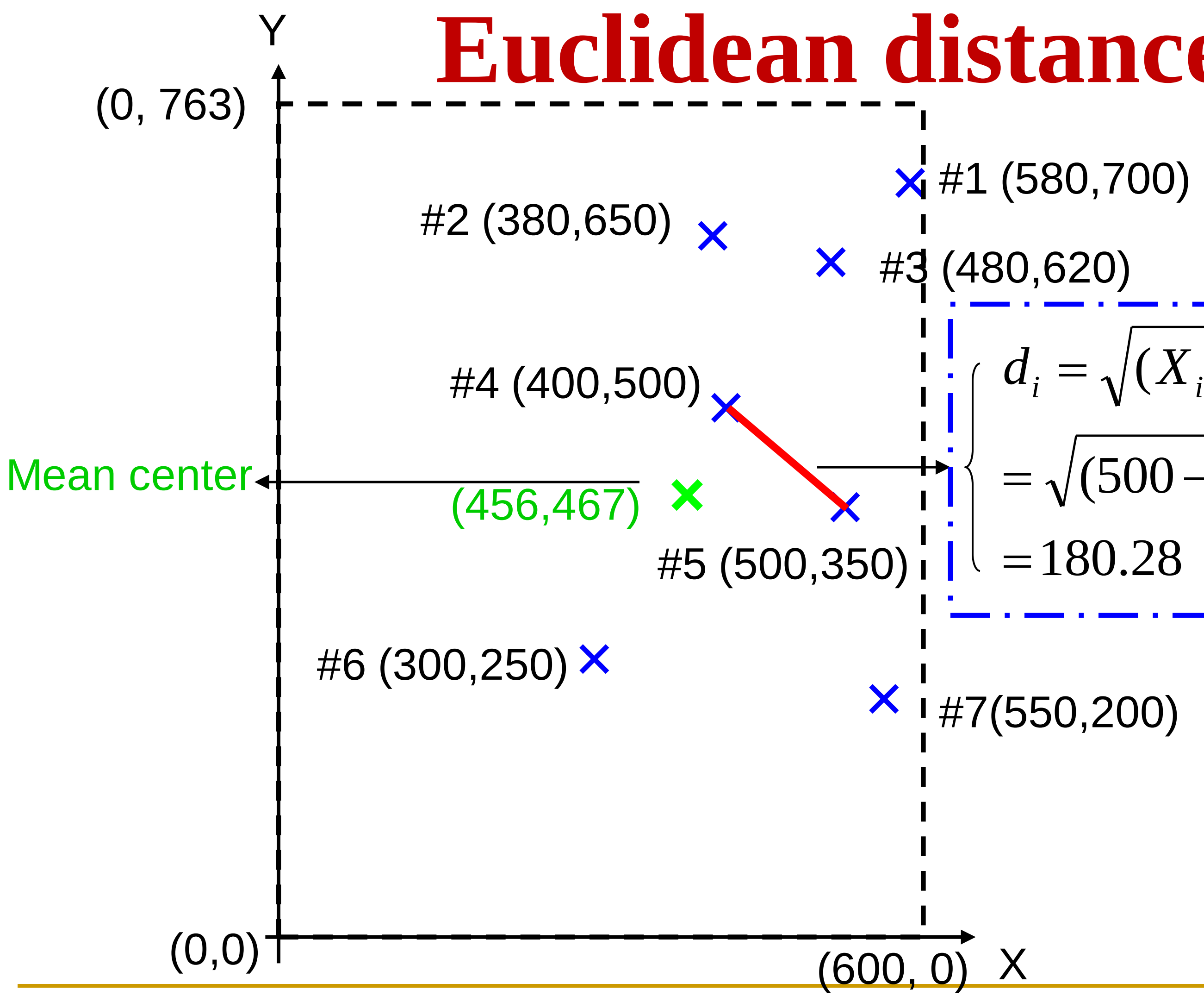
Summary

- Global Moran's I and local I_i have different equations, one for the entire region and one for a location. But for both of them (I and I_i), or the associated scores (Z and Z_i)
  - **Big positive values →positive spatial autocorrelation**
  - **Big negative values →negative spatial autocorrelation**
  - **Moderate values → random pattern**



Network Analysis: Shortest routes

Euclidean distance



$$\begin{cases} d_i = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} \\ = \sqrt{(500-400)^2 + (350-500)^2} \\ = 180.28 \end{cases}$$

Manhattan Distance

- Euclidean median
  - Find (X_e, Y_e) such that

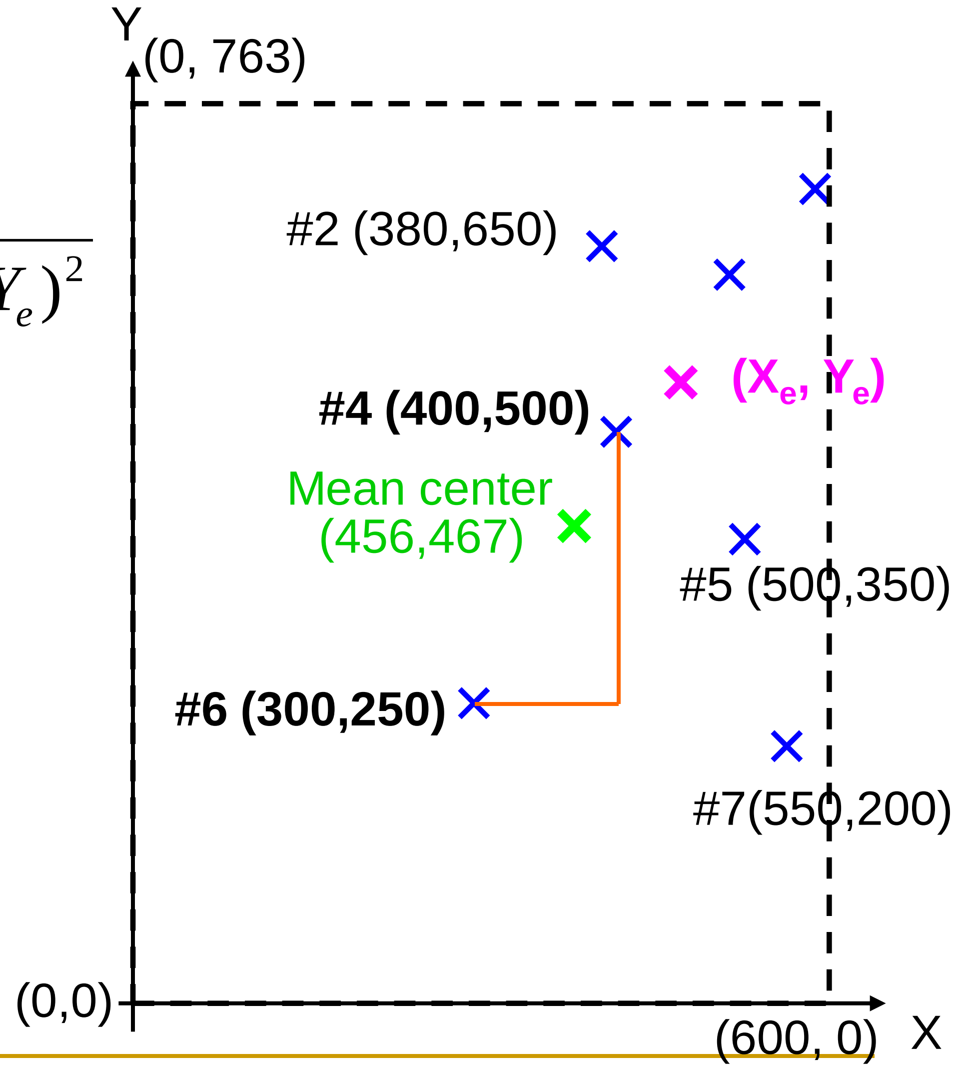
$$d_e = \sum \sqrt{(X_i - X_e)^2 + (Y_i - Y_e)^2}$$

is minimized
  - Need iterative algorithms
  - Location of fire station
- Manhattan median

$$d_{ij} = | X_i - X_j | + | Y_i - Y_j |$$

$$= | 400 - 300 | + | 500 - 250 |$$

$$= 350$$



Summary

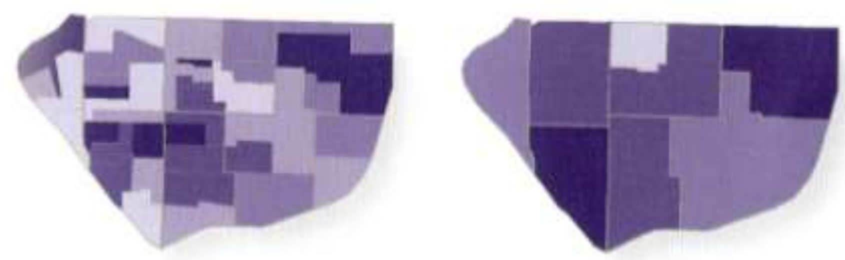
- What are spatial data?
- Mean center
- Weighted mean center
- Standard distance
- Weighted standard distance
- Euclidean median
- Manhattan median

Calculate in GIS environment

Spatial resolution

- Patterns or relationships are scale dependent
  - Hierarchical structures (blocks → block groups → census tracks…)
  - Cell size: # of cells vary and spatial patterns masked or overemphasized
- How to decide
  - The goal/context of your study
  - Test different sizes (Weeks et al. article: 250, 500, and 1,000 m)

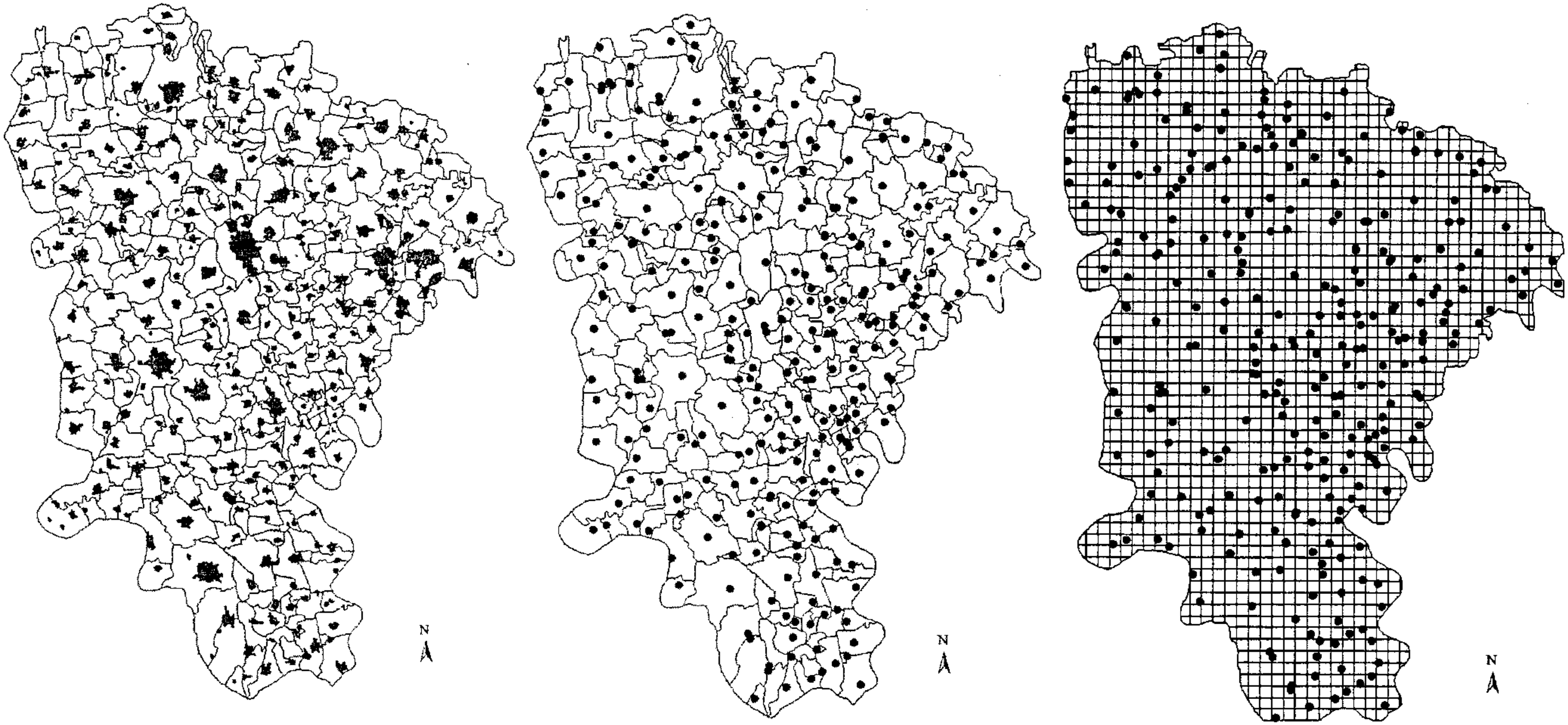
Vegetation types at large (left) and small cells (right)



% of seniors at block groups (left) and census tracts (right)

Administrative units

- Default units of study
  - May not be the best
  - Many events/phenomena have nothing to do with boundaries drawn by humans
- How to handle
  - Include events/phenomena outside your study site boundary
  - Use other methods to “reallocate” the events /phenomena (Weeks et al. article; see next page)



A. Locate human settlements using RS data

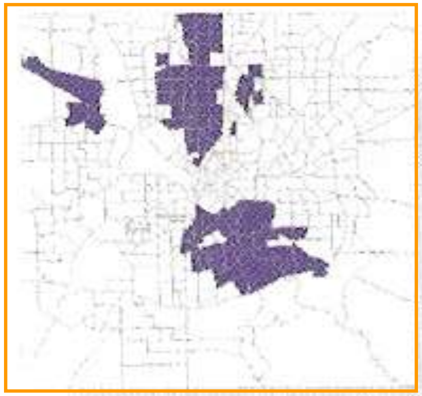
B. Find their centroids

C. Impose grids.

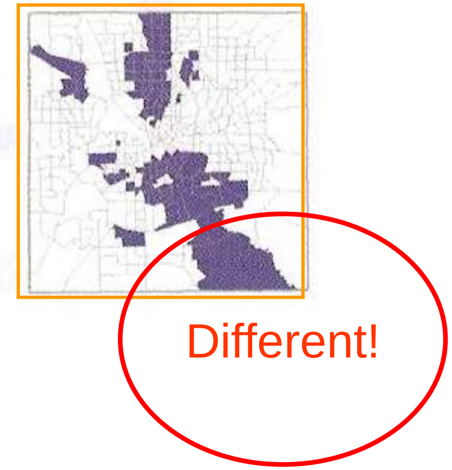
Edge effects

- What it is
  - Features near the boundary (regardless of how it is defined) have fewer neighbors than those inside
  - The results about near-edge features are usually less reliable
- How to handle
  - Buffer your study area (outward or inward), and include more or fewer features
  - Varying weights for features near boundary

a. Median income by census tracts



b. Significant clusters (Z-scores for I_i)



c. More census tracts within the buffer (between brown and black boxes) included

d. More areas are significant

Applying Spatial Statistics

- Visualizing spatial data
  - Closely related to GIS
  - Other methods such as Histograms
- Exploring spatial data
  - Random spatial pattern or not
  - Tests about randomness
- Modeling spatial data
  - Correlation and χ^2
  - Regression analysis